Visualization of the Vortex Dynamics in a Type-II Superconductor in a Periodic Dissipative State

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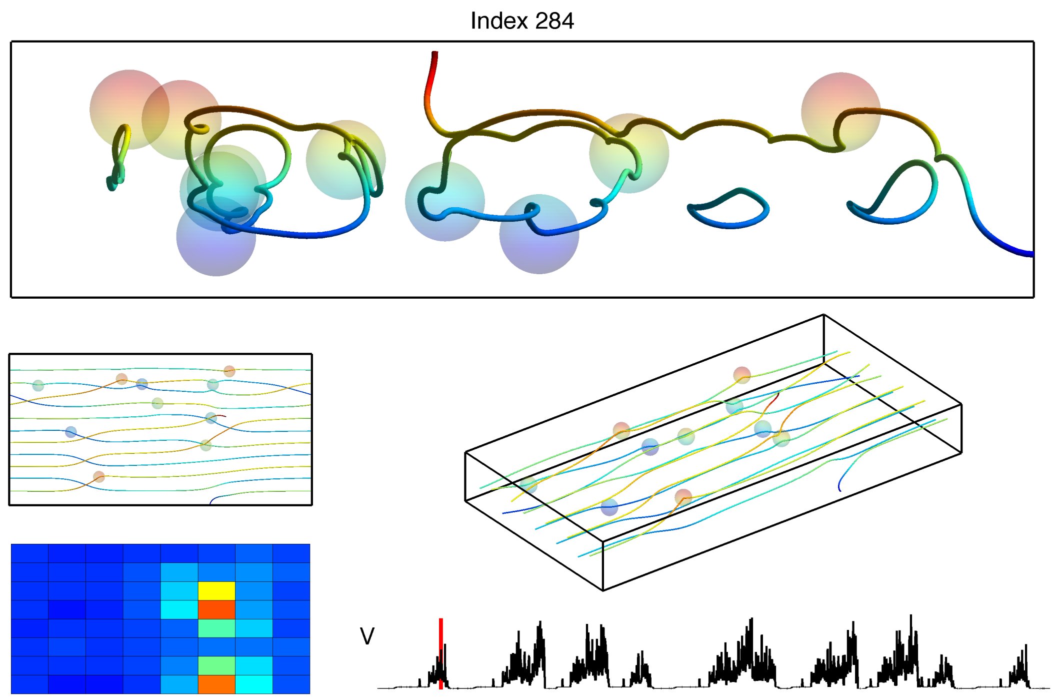

Figure 1: Single frame of the movie corresponding to a snapshot of the vortex state. At the top, the slab of the superconductor is shown along an x-axis view. The middle has a z-axis and isometric view. On the bottom left, a measure of local information entropy change indicates that where the vortex has attached to the surface of the superconductor, the vortex is moving very quickly (red) while the rest of the system is not changing at all (blue). On the bottom right, a time sequence of the voltage response of the superconductor indicates the current frame with a red line.

Abstract

Using state-of-the-art analytic techniques for extracting a singularity from a complex field, we can visualize the vortices in the largest time-dependent Ginsburg-Landau simulations of a superconductor performed to date. Our visualizations reveals the internal dynamics of the vortices in the superconductor, explaining the cause of a voltage spike in the electromagnetic response of the system. These simulations and visualizations indicate the emergence of a periodic dissipative state in a superconductor under certain external magnetic field and current conditions and explain a curious electromagnetic response that has been observed in experiments.

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1 Introduction

Superconductors, materials that can conduct current without any loss, are used in applications ranging from MRI machines to particle accelerators. Materials scientists are interested in understanding and controling the complex dynamic properties of superconductors. Designing superconductors that can sustain higher lossless, or critical, currents at higher temperatures could lead to technological advances affecting low-cost power transmission in the electrical grid, computing technology, and improved electromagnets.

In a type-II superconductor, the dynamics of magnetic flux vortices fundamentally determine electromagnetic response of the material. While the superconducting properties of a type-I superconductor break down catastrophically when the external magnetic field is too high, for a type-II superconductor the breakdown is more graceful. Above a critical level, an externally applied magnetic field penetrates the system in the form of flexible flux tubes, or vortices, which carry integer numbers of flux quanta (typically one flux quantum). The magnetic flux in the vortex core is screened by a circular supercurrent around it. When the vortices move, the system becomes dissipative; and a finite voltage drop across the system, corresponding to resistance, is observed. Thus the behavior of the vortices is an important determinant of the performance of the material. Material defects, or inclusions, distributed through the type-II superconductor can trap the vortices, pinning them in place and allowing the material to sustain a higher current.

Recent experiments on a type-II superconductor composed of molybdenum-germanium[5] revealed an interesting and unexplained effect: if a magnetic field is applied in a parallel direction with an external current, the superconductor shows a peculiar behavior upon increasing the field. Initially, the magnetoresistance increases with the magnetic field, as expected. But then at an intermediate field level, the resistance decreases to zero. As the magnetic field increases further, the resistance reappears and again increases with the increasing magnetic field.

We use a large-scale time-dependent Ginzburg-Landau (TDGL) model of the superconducting material to reveal the underlying mechanism of this resistance suppression. While computationally intensive, TDGL simulations are an attractive method because they correctly capture the vortex dynamics. Using the TDGL simulations, plus novel methods for analyzing the simulation data, our visualization movie shows how a periodic dissipative vortex state emerges at an intermediate-level magnetic field. One frame of the visualization movie is shown in Figure 1.

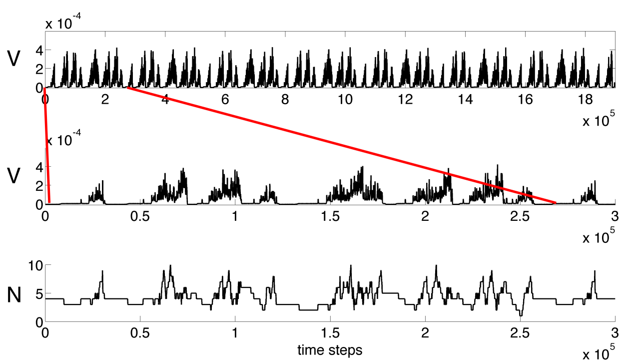

Figure 2: (Top) Voltage vs time step output of a simulation at an intermediate level magnetic field. (Middle) An expanded view of of one voltage cycle and the beginning of the next cycle. (Bottom) Total number of vortices measured in the simulation during the cycle.

2 Superconductor Simulations

Large-scale models of superconductors are being developed to predict and control the macroscopic behavior of these materials by designing the landscape of inclusions inside a type-II superconductor. These models use TDGL equations to solve for the time evolution of a complex-valued field ψ and the electromagnetic vector potential $\mathbf{A}$. A pair of coupled partial differential equations, the TDGL equations are initialized on a structured mesh and integrated forward in time by using a finite difference method.

A molybdenum-germanium type-II superconductor was modeled as a long thin slab by using a structured mesh of 256x128x32 nodes. The simulation of the slab is periodic in the x-direction but has a no-current boundary condition in the $y-$ and z-directions. A small external current and magnetic field was applied in the x-direction. An ensemble of simulations was distributed over a GPU cluster varying the density of inclusions in the material, the applied current, and the the external magnetic field. Over a certain intermediate parameter range, a curious voltage response of the system is observed. The voltage spikes and then drops back to approximately zero (see Figure 2). The pattern of the voltage response is periodic, repeating a set pattern of spikes over and over. As the strength of the magnetic field increases, the time-averaged voltage also increases and then decreases, matching the experimentally observed trend. To understand the mechanisms causing this periodic dissipative state, we need to visualize the behavior of the vortices in these simulations.

3 Visualizing Vortices

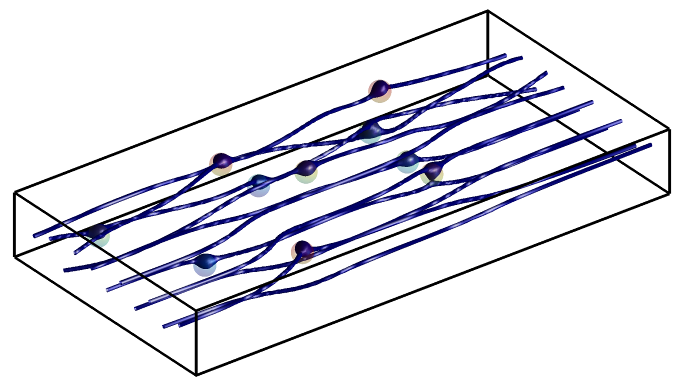

Figure 3: Traditional method of finding vortices by generating isosurfaces of the magnitude of the order parameter field. This method obscures the behavior of vortices inside inclusions.

Vortices are an implicit feature of the complex-valued order parameter field ψ. In the past, vortices have been visualized in TDGL simulations by examining contour plots and isosurfaces of the magnitude of ψ [1, 8, 7, 2, 4]. Figure 3 shows an example isosurface generated from the simulation data. In the image, the inclusions have been added and depicted as transparent spheres. Isosurfaces have been traditionally used because they are easy to construct and easy to render with graphics hardware. Using isosurfaces, however, blurs the fine details of the vortices. Because inclusions in the TDGL simulations are modeled as suppressions of the magnitude of ψ, when a vortex enters an inclusion, the vortex is obscured. When two vortices enter the same inclusion, the details of their interaction inside the inclusion are lost.

We have developed a feature extraction technique for exactly finding the vortex core lines from the phase field of ψ. Previously, aspects of this technique have been applied to trace vortices in a small-scale data [11, 9] and to find singularities in experimentally generated data from optical fields [10, 3]. However, the target and scale of our application, the techniques for unwrapping the phase locally, and the further analysis permitted by the precision of our calculations are unique to our work. With our method, the vortices can be easily described at a resolution even finer than the mesh itself and thus distinguished even inside an inclusion. Each vortex is an object represented by an ordered sequence of closely spaced points, each of which lies on the exact center line of vortex. The precise determination of the vortex cores allows the interplay of the vortices inside a model superconductor to be visualized in higher resolution than previously possible. This feature extraction method also massively reduces the data representation of a vortex and provides a simpler representation for further analysis and feature tracking from frame to frame.

Briefly, vortex objects are extracted from a field defined over a structured mesh at a given time step by locating all mesh element faces punctured by vortices. Matrix operations representing a gauge transformation and closed loop integration around every mesh element face are performed on all planar slices of the mesh. If the mesh element face contains a singularity in the phase field, then the exact point the vortex punctures the mesh element face is determined by interpolation. By tracing through the set of punctured mesh elements, an ordered set of points describing a single vortex object is generated. This vortex-finding algorithm was implemented in Python. The matrix calculation and linear algebra were performed by using the *numpy* library, which uses the BLAS and LAPACK packages. A Matlab script was then used to visualize the vortices. To aid visibility, the vortices were given an arbitrary radius and rendered as thin tubes. In the movie, the thin superconductor slab is shown from two angles, looking down the z-axis and looking along the x-axis of the slab. The vortices are colored by height in the z-direction. From the z-axis view, this coloring makes it apparent when the vortices do and do not intersect.

The movie clearly shows that even during the voltage spike, most of the length of each vortex is barely moving while a small part travels rapidly. To highlight where the vortex movement is localized, we additionally visualize the change in local information entropy, using the Information Theory Library framework (ITL) [12]. At each frame, the difference in the magnitude of ψ at each mesh point is calculated relative to the previous frame. The mesh is divided into 64 blocks in the $x-y$ plane. A histogram is calculated of the differences for each block, and then the entropy of the histogram is calculated. If locally the vortex moves very little, or if all the movement is inside an inclusion, then a low entropy is calculated. If the vortex moves swiftly, a high entropy is calculated. A calculated entropy from 0.0 to 5.1 is displayed as dark blue to red.

4 Visualization Movie

The movie created over a single voltage cycle of Figure 2 reveals the internal vortex dynamics of the periodic dissipative state. Frames of the movie were generated every 100 simulation time steps. A single frame of the movie is shown in Figure 1, and key frames of the x-axis view are shown in Figure 4. Evident from the z-axis view, the vortices are stretched like taut spaghetti noodles in the direction of the magnetic field. Since vortices cannot terminate except at the superconductor surface, the vortices in the simulation tend to cross the x-boundary and form closed loops, which is most evident from the x-axis view. At the beginning of the cycle, four vortex loops wrap through the slab eleven times.

Attracted to inclusions, vortices bend to pin themselves on a nearby inclusion. If the vortices were perfectly straight in the x-direction, the external current applied along the x-axis would impose no force on them. The bending of the vortices induces a slight Lorentz force from the current, pushing the vortex along the y-axis. In the first 100 frames of the move, each vortex slowly stretches in the y-direction where it is bent.

Two of the vortices are pinned in the same inclusion. From frame index 1 to 97, the two vortices bend slowly toward each other inside the inclusion. At frame index 101 they recombine with each other. That is, they cut and reconnect locally. The recombination occurring inside an inclusion happens very slowly. At frame index 188, two other vortices “phase slip” and recombine outside an inclusion. Recombinations outside of inclusions occur extremely fast. The corresponding block in the information entropy plot shows that a rapid change in the field occurred localized around the recombination location.

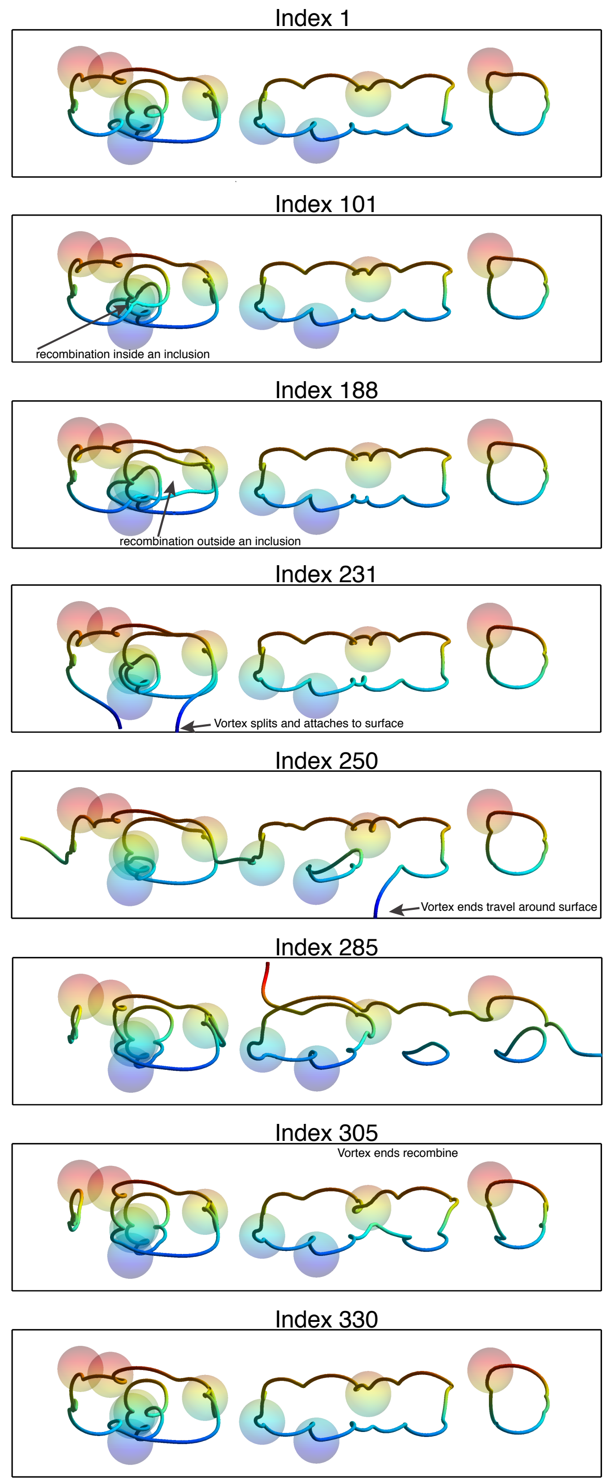

Figure 4: Key frames of the x-axis view of the slab during the first spike of the voltage cycle. The intense part of the voltage spike corresponds to the vortex ends attaching to the superconductor surface and traveling around the system until they reattach.

At frame index 231, a vortex loop splits at an inclusion that is close to the boundary, and two ends attach to the boundary of the system. The two ends of the vortex attached to the boundary are now pushed helically around the system in opposite directions by the Lorentz force. We note that the vortex splits and moves along a cross section of the system where several inclusions are roughly aligned in the y-direction. In the x-axis view, we can see that the vortex ends jump and recombine from x-aligned vortex to x-aligned vortex like a zipper as they travel. From the ITL plot, we can tell that for this voltage spike, almost all the motion was confined to a small cross-section of the system. Eventually the two ends have traveled approximately the same distance around the outer circumference of the slab; they meet and join, and the the system relaxes back into a quiescent state of four loops. At frame index 330, the system is in approximately the same state at frame index 1.

The middle of Figure 2 depicts the voltage spikes observed over the cycle. The intense voltage spike occurs when the ends of the vortex attach to the slab surface and are driven helically around the system. The bottom of Figure 2 counts the number of vortices in the system. The rapid series of recombinations as the two ends traveled are indicated by the rapid changes in the number of vortices present in the system.

5 Conclusion

Our visualization shows that the periodic dissipative state observed in a simulation of a type-II superconductor correlates with vortex splitting, attaching to the boundary, and then traveling helically around the boundary until it finally reattaches. The vortices, stretched by the magnetic field, bent by the inclusions, and pushed by a Lorentz force when bent, occasionally approach each other close enough to phase slip, or close enough to the boundary to detach, setting off a cascade of motion. These simulations and visualization explain a electromagnetic response observed in experiments on a type-II molybdenum-germanium superconductor and are the first time that we have been able to observe the internal dynamics of the material.

The simulations of superconductors shown here is part of an effort to implement large 3D simulations where macroscale phenomena can be observed [6]. Reaching the macroscale in these large 3D simulations will require advanced HPC techniques and resources. It will also require the codesign of data analysis and visualization techniques that can scale with the application as the projected memory required to store the entire state of the simulation grows. By codesigning data analysis and visualization alongside the development of increasingly larger HPC simulations, scientific discovery can keep pace with the generation of data.

Acknowledgments

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